The liar's paradox and other philosophical absurdities

Alan Saunders: Hi, this is the Philosopher's Zone, I'm Alan Saunders, and what I am now saying is false. Bit of a problem there. If I tell you that what I am now saying is a false, and it is false>/em>, then what I am now saying is true. But if it's true, then to say that it's false, is false. We're going round in the circles known in philosophical circles as 'The Liar Paradox'.

The Liar Paradox gives its name to This Sentence is False, a new book by Peter Cave who teaches philosophy in the UK and the Open University and City University, London. So can he tell us exactly what a paradox is?

Peter Cave: Yes, a philosophical paradox normally starts off with some very, very commonsensical assumption or belief which we just think is obviously true. And then we do a little bit of reasoning, which seems so incredibly simple, so incredibly straightforward, that that can't be going wrong, and yet then we reach a conclusion which is against all commonsense belief. So either we have to accept the conclusion - which is very, very odd for us, because it goes against everything which we believe - or we have to say our starting point, our premises, our assumptions are wrong, but again they are normally very, very straightforward, or we have to find something wrong with the reasoning. And so why philosophical paradoxes are very, very deep I think, often, is because when we examine the premises, when we examine the reasoning, when we look at the conclusion, we just can't sort out exactly where something has gone wrong. Why that is very, very important I think is because it's showing that the concepts which we use in understanding the world, our very comprehension and representation of the world, has some kinks in it, something goes wrong. We don't really think the world itself is paradoxical, is contradictory, yet when we represent the world we seem to get paradoxes or contradictions. So that's why it strikes me as being very, very deep and important.

There's one other reason why I think it' s very useful, namely it generates a certain humility amongst philosophers. I think sometimes philosophers have tended to think 'Oh, we can sort everything out', whereas I like to remind them there are these paradoxes and so we should be a little bit humble before the universe.

Alan Saunders: There's an alleged paradox that I don't really think is a paradox at all -

Peter Cave: And that's not because you've drunk too much, I hope, Alan.

Alan Saunders: No, I don't think that's the reason. It's the Barber paradox. Tell us about that.

Peter Cave: This again is a traditional paradox. It's

Bertrand Russell who brought it up more recently (by recently I mean 100 years or so ago: philosophers live in the past, in a sense) . And the paradox is you just say, Hey, there's a barber in a certain village, say the barber's in Alcala, and he shaves all and only those people who do not shave themselves.' And in that case, well yes, obviously you can shave those people who do not shave themselves, if you think that he's shaving Albert and Brendan and Clarissa and so on (I assume Clarissa needed to be shaven in that example) and he shaves only such people, but the problem comes of course is: does he shave himself? Because if he is somebody who shaves himself, then he ought not to be shaving himself and if he is somebody who isn't shaving himself, then he should be shaving himself because he's the barber. And so we have a contradiction about the Barber of Alcala.

Alan Saunders: Well we have a contradiction, but it seems to me that we have there what philosophers refer to as a reductio ad absurdum, where you assume the truth of something and then as a result of assuming the truth, you derive a contradiction. It's not a paradox because all you're doing is proving that no such barber can exist.

Peter Cave: Indeed. Many people, many philosophers, including myself would say with this particular traditional paradox, we do have an easy solution here because as you've just rightly said, Alan, being a philosopher yourself, we can quickly say, Well this just shows that such a barber cannot exist. And I suppose I'd say perhaps it shows that such a barber can exist but we have misdescribed him by saying he could be someone who shaves all and only those individuals who do not shave themselves in Alcala. And so there are similar pseudo-paradoxes we may think of them, which can generate a similar problem. One which I particularly like is the liberated secretaries one, in which we say, Hey, there are some clubs (and this is true) in which there's a secretary of each club but they are not allowed to be members of those clubs themselves. So the secretaries get a bit miffed about this and so they think, Ah, we'll set up our own club and the club is for all and only those secretaries who work for clubs for which they are not allowed to be members themselves. And all goes well, and that's perfectly coherent; there's no contradiction there. No problem at all. Secretaries could set up a club only for secretaries who cannot join the clubs for which they work. And as you can anticipate, Alan, the problem comes when the club is so successful that they themselves employ a secretary. Big problem. I mean here we have a secretary who belongs to a club; should they allow her to join the club, or not? If she joins a club, then she does belong to a club and works for a club at the same time, and so she ought not to be allowed to join, but equally so if you go the other way around, then she should be allowed to join. You have a contradiction. And yet we could set up such a club and there could be a secretary of such a club, so that starts making us think we have to be a little bit more careful how we analyze these problems.

Alan Saunders: Well in the case of the secretaries, presumably what you just say, I mean if you're at the committee meeting of the club, you say, Well, if we let the secretary be a member of the club, we're going in this once instance, and only in this one instance, to break our own rules, and that's not difficult is it?

Peter Cave: Well, it does point out that what seemed to be perfectly acceptable rules in the first place do in fact generate a contradiction under certain possibilities. For example, in this case, under the possibility of employing a secretary which the club rule founders hadn't thought about. And I think you find this sometimes in legal constitutions of countries , that lurking somewhere there may well be some circumstances such that they'll suddenly find a contradiction arises if they're not careful. As I'm sure you know, Alan, in fact these lead into Russell's paradox and so on, which are concerned with abstract entities called sets, but I think you were discussing that type of problem in a previous program. I think the mathematical example perhaps can be seen as a manifestation of the far more general point which can then be manifested by the simpler example, namely that of Zeno's Paradox. If I may quickly run through that? To relate it?

Alan Saunders: Yes, please do.

Peter Cave Some of your listeners may well be aware of this paradox. It's a wonderful paradox from Zeno, an ancient Greek philosopher, about 450BC, and one particularly colourful example of it is one in which Zeno says, Hey, Achilles is the fastest runner in Athens, and here we have a slow tortoise. Let them have a race. But because the tortoise is so slow, to be a little bit fair let's give it big start, say anachronistically, a 100 yards. But the tortoise is game for this, perhaps there's a big lettuce leaf to give an incentive for the tortoise to run the race, and perhaps that's an incentive for Achilles, and that's not quite so plausible. But anyway, so Achilles and the tortoise set off, but the tortoise has got a head start of 100 yards, and of course Zeno reasons thus, saying, Well, before Achilles can win the face, he must overtake the tortoise, and before he can overtake the tortoise, he must manage to get to where the tortoise is, where it starts. And so of course Achilles runs to where the tortoise has started, but by which time the tortoise will have moved a little bit further ahead, and so in order for Achilles to win the race, he must now get to where the tortoise is now and so by the time he gets to that position, then the tortoise will have moved a little bit further still, and so on, and so on. And so Zeno drew the conclusion that therefore it's impossible for Achilles to win the race, and yet of course we know from empirical evidence that Achilles does win the race. Zeno, though, was inclined to say, No, no, no, you know, I'm such a conceited philosopher and my reasoning is perfect: it must be there's something wrong about the world, not about my reasoning. In some sense or other, motion is illusory. And there are many, many simple examples of this type of problem, so if you just look at the wall opposite you and think, Well, before I can reach the wall opposite, I must walk half the way, and half the way, and half the way, that is endless series: a half, plus a quarter, plus an eighth, plus a sixteenth, and that series goes on endlessly, and apart from some mathematicians, and mathematically inspired philosophers, some of us still worry about that type of argument.

Alan Saunders: Well, yes, but given that as we know -

Peter Cave: I notice the Royal 'we' there. I believe you're not yet a Republic.

Alan Saunders: Yes. Given as we know that Achilles is going to win the race and given that I can reach the wall, and if I don't want to embrace the rather far-reaching conclusions of Zeno, there must be something wrong with the way I'm thinking about these things, mustn't there?

Peter Cave: I accept that, yes, but exactly where does it go wrong? Mathematicians say, no, no, no, this isn't a problem at all, because we know some infinite series, some endless series, are convergent and so the infinite series of, for example, a half plus a quarter plus an eighth does end up being at one, and so they say the sum of that series is indeed one, and they can work out exactly when in any particular example Achilles would overtake the tortoise and win the race. I think some of us, though, would say, Yes, we know the mathematics works, we're not challenging that, but nonetheless there is a conceptual problem here about how to make sense of completing an endless series. And there is some kink in our conceptual scheme in trying to give an understanding of that.

Alan Saunders: I suppose that these paradoxes of infinity are related to the paradox created by the hotel that was set up by the German mathematician, David Hilbert -

Peter Cave: Indeed, a very, very eminent mathematician.

Alan Saunders: Indeed. Tell us what goes on in the hotel.

Peter Cave: Well, it casually begins, which you can see I'm already pointing over some scepticism to the story, but it casually starts of saying, Hey, we have an Infinite Hotel, by which we mean it's a hotel with an infinite number of rooms, numbered 1, 2, 3, 4, 5 and so on without end. All of the rooms are occupied. Now in a regular sort of hotel, if all of the rooms are occupied, then if a new guest arrives you have to turn him away. But, argued Hilbert, not so with the Infinite Hotel because if a new guest arrives, even though the Infinite Hotel all the rooms are taken up, we can still accommodate the new guest who has arrived. How do you do this? Well we ask the guest in room No.1 to move into room No.2, the guest in room No.2 to move into room No.3, and so on and so on. Because there's an infinite series, any particular guest in any particular room can always move on to the next one, because there's no end to the series. And so you have freed up room No.1 and hence the guest can be accommodated. Indeed, Hilbert points out, which quickly becomes obvious, that even if an infinite number of guests arrived, and the hotel was already full, you could still accommodate them because you could move all the odd number guests into the even numbers and that would release all the odd numbered rooms for the infinite number of guests, new guests, who have arrived, and of course there's an infinite number of odd numbers, just as there's an infinite number of even numbers. Paradoxically there's as many, say, even numbers as there are even and odd numbers. You could match them up.

Alan Saunders: You mentioned in your book that we're often told this without the batting of an eyelid.

Peter Cave: I think we should bat.

Alan Saunders: I was going to say you do think that some batting is in order Why is that?

Peter Cave: Well , because it does so casually start off with an Infinite Hotel, which we know of course cannot exist, and also it starts off saying all of the hotel rooms are full. And so there's already a tension in the story, isn't there? The way in which Hilbert gets it going is by not stressing the 'all' part of the story, but of course if we started off saying all of the rooms are occupied, then we suddenly are baffled as to how we can accommodate a new guest. I think once again it's an example of how the abstract realm, what we can make perfectly good sense of in mathematic s, in trans-finite mathematics and so on, is a wonderful thing and can often frequently in fact help us in understanding the real world around us, and helping science and so on. But nonetheless to think there's an easy crossover is to my mind very, very iffy.

An example which derives from the great Ludwig Wittgenstein: he gives the example that, consider a chess game, it could be any game, any board game, for which the piece, say the Knight, is one in which you can move it two squares up and then one square to the left or right, i.e. there's a mixture of moves there. With such a move, yes, a Knight can make that move, but in the games of chess it's not allowed for the Knight to make just half a move, or a quarter of a move or a third of a move. There are no half moves or quarter moves in chess, and yet the physical wooden piece which you move, could indeed make half moves and quarter moves, and so the understanding of the rules of chess do not easily play over to what goes on in the physical world. In chess there are no half moves, in drafts there are no half moves, but there are half moves when you move a bit of wood around. And so that's just a very simple little analogy to make us think how we describe different situations may not move over and may not be representative very easily in the empirical world.

Alan Saunders: On ABC Radio National you're with The Philosopher's Zone and I'm talking to Peter Cave about his book The Sentence is False: an introduction to philosophical paradoxes.

Peter Cave: We all have crosses to bear in talking to me. Mind you, it passes the time, but then as Samuel Beckett said, 'Time would have passed anyway'.

Alan Saunders: It would, indeed. Peter, you're talking about how we address the realities of the world and how our paradoxes may expose the limitations of our thinking, and one of the paradoxes which perhaps does that is the surprise examination or its slightly bloodier counterpart, the surprise execution. What's going on there?

Peter Cave: I must admit I do prefer the more ghoulish version, the surprise execution or the surprise hanging. it's often known as. But perhaps to be sensitive to sensitive Australians listening to this, perhaps we should stay with the examination. The story is - and of course this does actually happen in real life - is pupils may be told that in the coming week there's going to be a surprise examination. It's going to occur on Monday or Tuesday or Wednesday or Thursday or Friday. Those are the five days of the week. School is just a five-day week in this example. The pupils know about this and they say, I wonder what we mean by surprise, and the teacher is very, very clearly says, What I mean is very, very clear, namely on the morning of the examination you'll have no good reason, or you won't know that the examination is going to occur. Let's pretend the examination is bound to occur at midday, at noon, on any one of those days. And so suppose some bright kid, some bright pupil is sitting there at home on Sunday thinking Should I bother to revise or not? And then he starts doing a bit of logic, he's a bit like a Quine or a Tarski, a great logician , and he starts thinking, hold on, if I get through to Friday morning unexamined, then I would know that the examination would have to be on Friday lunchtime, Friday at noon, because the teacher always tells me the truth. And so clearly Friday must be ruled out. Friday's not an option for the examination day. So he's sitting there on Sunday thinking, Hold on, suppose I get through to Thursday without an examination, then if I do get through to Thursday, by my previous reasoning I know the examination cannot occur on Friday, and so it would have go occur at noon on Thursday, because the teacher always tells the truth. But then I'll know it was going to occur and so therefore it would not be a surprise, and so therefore the examination cannot occur on Thursday. And he reasons in a similar way, ruling out Wednesday, Tuesday and Monday, and so of course on Sunday he sits there complacently not bothering to revise because he thinks such an examination cannot be set and maybe on Tuesday, maybe on Thursday, who knows, his complacency is intruded upon and he suddenly gets examined by the schoolteacher. What has gone wrong with the reasoning? Over to you, Alan.

Alan Saunders: Well, I wonder what has gone wrong with the reasoning.

Peter Cave: I hasten to say by the way, there are probably hundreds of articles written about this of different attempted solutions and people carry on writing about them. One point is why ever assume the teacher must be telling the truth. But a related point would be to say talking about Fridays and Thursdays and so on, that's all white noise, that's just to distract us maybe. Suppose a teacher says, this morning, to the pupil , This afternoon there's going to be an examination, but you don't believe there's going to be an examination this afternoon. In other words, you've got the idea which the teacher's saying in the original case, namely, something's going to happen but you won't believe that thing's going to happen. Then, of course, we don't know what to believe because you've been given seemingly contradictory information. Although it's true that an examination may happen this afternoon, and I don't believe the examination will happen this afternoon, for me to express it in the first person saying, 'There will be an examination but I don't believe there will be an examination' is absurd. It's rather like saying, There will be an examination and there won't be an examination. And so this moves some of us into thinking that the surprise examination is really a particular modified version of what became known as Moore's Paradox. That's a paradox named after the great George Edward Moore, a Cambridge philosopher in the early 20th century, and he pointed out about how odd it is for say Peter Cave to say, I don't know, 'Kangaroos live in Australia, but I don't believe kangaroos live in Australia', or 'Kevin Rudd is Australia's Prime Minister, but I don't believe Kevin Rudd is Australia's Prime Minister'. That is absurd for Peter Cave to say, for me to say that, and yet of course it may well be true. It may well be true that Kevin Rudd is Australian Prime Minister, he is, and it may also be true that Peter Cave doesn't believe it, but that's a bit of information I cannot say about myself without absurdity, yet others can say it of me . And so I think the pupil is really in that rather difficult situation. The pupil can't say of himself, There'll be an examination but I don't believe it. But the teacher may have reasons to think that he's such a thoughtful pupil, whatever, that indeed there will be an examination, but he won't believe there will be one, or vice-versa for that matter.

Alan Saunders: I want to end on the very highest possible level with God.

Peter Cave: Why? Which one, by the way?

Alan Saunders: Well it probably has to be the Abrahamic god. It has to be a god who is omnipotent, omnipresent, omnibenevolent and so on, and there are various paradoxes arising out of believing in such a being, one of which is the old mediaeval one which takes the form of the question, Can God create a stone, which he can't lift? Now if you say, Yes he can, then you're suggesting that there is a limitation in God's power, in that there's a stone in the world that he can't lift, and if you say that he can't create such a stone, then you're similarly suggesting that there's a limitation in God's power, because you're suggesting that he can't create such a stone.

Peter Cave: What you're saying is true, except it's also false, to make it sound paradoxical. It depends of course on your understanding of omnipotence, on being all-powerful. And one very quickly would say, Well hold on, for something to be all-powerful, it doesn't mean it can do the logically impossible. So, for example, an all-powerful God cannot make it both rain and not rain at the same time in the same place, because that's a logical impossibility. To say, Hey, he's not that powerful if you can't say make two plus two equals five, would be like saying Hey, God's not that powerful because he can't blibble-blobble-bleable. We've got no idea what it is to blibble-blobble-bleable and we've got no idea what it is for at the same place at the same time, it's both to be raining and not raining. So the manoeuvre then is to say, Well, with regard to the immovable stone, the immovable stone, that concept, the concept of an immovable stone, is not itself self-contradictory, but the idea of there being an all-powerful being making an immovable stone, is a piece of self-contradiction. I think that perhaps is a little too quick, but that's the very quick way in which philosophers normally handle that particular problem but you might therefore draw the conclusion that either God can make an immovable stone, but we have to accept omnipotence means you cannot move it, or he can't make the immovable stone, but were there to be one you would be able to move it. You could equally well argue Well hold on, this shows there isn't a God, because there is an immovable stone somewhere in the universe, or there could be. So you can move it along in different directions.

Alan Saunders: The book is called: This Sentence is False, and I have been talking to the author, Peter Cave, who teaches philosophy at the Open University and at the City University in London. Peter, thank you very much indeed for joining us.

Peter Cave: And thank you for having me, as some good girls say.

Alan Saunders: Details of Peter Cave's book on our website, abc.net.au/rn/philosopherszone.

And if you head to this week's program and click on the spot that says 'Add Your Comment,' you can share a paradox or two with us and your fellow-listeners.

Thanks to producer Kyla Slaven, and sound engineer Charlie McKune. I'm Alan Saunders and I'll be back next week with another puzzling Philosopher's Zone.